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ORBITAL AND PERICENTRIC REFERENCE SYSTEMS IN THE SCHWARZSCHILD FIELD*

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Reference systems generalizing orbital and perigee coordinates systems, widely used in the Newtonian theory of gravitation for solving inertial navigation problems, are introduced in the Schwarzschild gravitational field using the tetrad formalism of the General Relativity Theory (GRT). Exact expressions are obtained for the angular precession velocity of the gyroscope in these reference systems in the case of its arbitrary geodesic motion, and the physical components of the dipole magnetic field intensity, which are used for the magnetic control of satellite orientation, are determined. It is shown that gravitational corrections of magnetic field intensity can, in principle, be measured by contemporary quantum magnetometers.

The main problem of satellite inertial navigation is the determination of physical phenomena directly in the accompanying reference system (RS) with subsequent conversion (whenever required) to any other RS /1,2/. In practice, a specific orientation of the measuring equipment in orbital motion in the gravitational field and its stabilization in the specified position during prolonged periods of time are required. Because of this, the respective RS (orbital, perigee, etc.) are of the form of moving coordinate systems with origin at the satellite center of mass and specifically oriented relative to the orbit plane. Thus in an orbital RS the first coordinate axis is directed along the instantaneous radius vector, and the second and third along the transverse and normal to the orbit plane, while in the pericentric RS the orbital orientation of axes is fixed in one of the pericenters of the orbit and remains unchanged along the trajectory /3,4/.

Owing to the increasing interest in the investigation of GRT effects with the use of satellites it is expedient to introduce in it the indicated RS concepts. In the theory of relativity the RS is defined by the Lorentz basis $e_{(k)}$ whose projections on the axes of a global system of coordinates x^μ are tetrads $h^\mu_{(k)}$ /5/. The indices denoted by letters at the beginning of the Latin and Greek alphabets assume the values 1, 2, 3, while those beginning from and including k and $\varkappa$ have the values 0, 1, 2, 3. Indices of quantities related to the Lorentz basis are enclosed in brackets. Hence the successive solution of problems of inertial navigation considered in the GRT necessitates the use of the tetrad formalism which enables us to introduce the RS with the required orientation of the three-dimensional triad $h^\mu_{(a)}$. In the GRT reference systems with triad vectors tangent to coordinate lines are often used. However this may introduce unimportant effects related to constant variation of the triad orientation along the trajectory, and impede the analysis of investigated phenomena.

The aim of the present investigation is the introduction using the tetrad formalism of RS accompanying /the satellite/ with orbital and pericentric triad orientation in any arbitrary geodesic motion of the reference body in the Schwarzschild field, and for determining in them of GRT corrections to the gyroscope motion and the measured components of the dipole magnetic field, since gyroscopes and magnetometers are generally used for orienting and stabilising satellites. It is necessary to take into consideration all possible perturbations affecting the gyroscope, and in the case of magnetic orientation control it is necessary to know components of the measured magnetic field as functions of orbital parameters (see, e.g., /3, 4,6/).

1. Let the reference body of the RS accompanying the satellite moves along a geodesic in the Schwarzschild field with metric

$$g_{\mu\nu} = \operatorname{diag}(-a^{-2},\ a^2,\ r^2,\ r^2\sin^2\theta),\quad a = (1 - 2m/r)^{-1/2}$$

We define its 4-velocity components as follows:

$$u_\mu = (-\varepsilon,\ aA,\ -\nu,\ h),\quad A^2 = \varepsilon^2 a^2 - 1 - H^2/r^2,\qquad H^2 = \nu^2 + h^2/\sin^2\theta \tag{1.1}$$

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The meaning of constants ε, H and h is that of total specific energy of the test mass, its orbital moment, and of its projection on the axis $\theta = 0$, respectively. The signs of ν and h are determined by the initial conditions.

We introduce in the orbital plane the polar system of coordinates (r, Φ) and define angle Φ by the relation

$$r^2 \Phi^{\cdot} = H, \; \Phi^{\cdot} = d\Phi/ds = (\theta^{\cdot 2} + \sin^2\theta\varphi^{\cdot 2})^{1/2} \tag{1.2}$$

Solution of the equations of motion by the method of quasi-conical section /7,8/ yields for the orbit an equation of the form

$$r = pw^{-1}, \quad w = 1 + e\cos\Psi(\Phi) \tag{1.3}$$

where e and p are the eccentricity and focal parameters, and the true anomaly Ψ is implicitly defined by the quadrature

$$\Phi = \int\limits_0^{\Psi} \frac{d\Psi}{F(\Psi)}, \quad F(\Psi) = \left(1 - \frac{4m}{p} - \frac{2m}{p} w\right)^{1/2} \tag{1.4}$$

The constants of motion ε and H are related to orbital parameters by formulas

$$\varepsilon^2 = \frac{\varkappa^2}{p\delta^2}, \quad H^2 = \frac{h^2}{\cos^2 i} = \frac{mp^2}{\delta^2}, \quad \varkappa = \sqrt{(p-2m)^2 - 4m^2e^2}, \qquad \delta = \sqrt{p - m(3+e^2)} \tag{1.5}$$

where i is the angle of inclination of the orbit plane to the equator.

Solution of the system of equations $g_{\mu\nu} = h_\mu^{(k)} h_\nu^{(n)} \eta_{(k)(n)}$, where $\eta_{(k)(n)} = \mathrm{diag}\,(-1, +1, +1, +1)$, that determines the tetrad $h_{(k)}^\mu$ requires further conditions (calibration /5/) that determine properties of the RS motion (calibration of the monad $h_{(0)}^\mu$)), and of the orientation of its three-dimensional basis vectors (calibration of the triad $h_{(\alpha)}^\mu$). Calibration conditions are specified on the basis of various physical, geometric and other considerations /5/. However, since the selection of calibration defining the orbital and pericentric orientation of the triad (particularly in the case of concomittance) is not obvious, we shall first construct tetrads of the required orientation for defining the RS fixed in relation to the global coordinate system. In this case the monad calibration is of the form $h_{(0)}^\alpha = 0$ and defines the tensor

$$\gamma_\nu^{\ \mu} = \delta_\nu^{\ \mu} - h_{(0)}^\mu h_\nu^{(0)}, \quad \gamma_0^{\ \mu} = \gamma_\mu^{\ 0} = 0, \quad \gamma_\beta^{\ \alpha} = \delta_\beta^{\ \alpha}$$

that projects the 4-vectors into the 3-space orthogonal to the monad and proper for the RS.

We introduce at every point of the orbit a triad whose orientation coincides with that of the orbital RS used in the Newtonian theory of gravitation, taking into account that the three-dimensional velocity vector $v^\mu = \gamma_\nu^{\ \mu} u^\nu$ and the three-dimensional vector $k^\mu = \gamma_\nu^{\ \mu} \delta_1^{\ \nu}$ in the direction of the present radius vector of the test mass lie in the orbit plane. We select k^μ as the first, and $m^\mu = \gamma^{-1/2}\varepsilon^{\mu\lambda\sigma} v_\lambda k_\sigma$ and $n^\mu = \gamma^{-1/2}\varepsilon^{\mu\lambda\sigma} k_\lambda v_\sigma$ as the second and third vectors of the space basis. Here $\gamma = \det \gamma_{\alpha\beta}$, $\varepsilon^{\mu\lambda\sigma} = h_{\tau(0)}\,\varepsilon^{\tau\mu\lambda\sigma}$, where $\varepsilon^{\tau\mu\lambda\sigma}$ is the Levi-Civita symbol. With this definition vector m^μ is oriented in the direction of motion and lies in the orbit plane, and n^μ is orthogonal to it. For the fixed orbiting tetrad, after normalization of the introduced vectors, we obtain

$$\begin{aligned} h_{(0)}^{\ \mu} &= (a, 0, 0, 0), \quad h_{(1)}^{\ \mu} = (0, a^{-1}, 0, 0) \\ h_{(2)}^\mu &= \left(0, 0, -\frac{\nu}{Hr}, \frac{h}{Hr\sin^2\theta}\right) \\ h_{(3)}^\mu &= \left(0, 0, -\frac{h}{Hr\sin\theta}, -\frac{\nu}{Hr\sin\theta}\right) \end{aligned} \tag{1.6}$$

Using the invariant determination of the angle we obtain the matrix of directional cosines of angles between vectors of tetrad (1.6) and vectors tangent to coordinate lines of the spherical coordinate system

$$\begin{aligned} \|\cos\eta\| &= \begin{Vmatrix} E & 0 \\ 0 & \Lambda \end{Vmatrix}, \quad E = \begin{Vmatrix} 1 & 0 \\ 0 & 1 \end{Vmatrix} \\ \Lambda &= \frac{1}{H}\begin{Vmatrix} -\nu & -h/\sin\theta \\ h/\sin\theta & -\nu \end{Vmatrix} = \frac{1}{\sin\theta}\begin{Vmatrix} -\sin i\cos\Phi & -\cos i \\ \cos i & -\sin i\cos\Phi \end{Vmatrix} \end{aligned} \tag{1.7}$$

which, after conversion to Cartesian coordinates, is usually applied in the Newtonian theory of gravitation for the introduction of orbital RS /3,4,6/.

Acting on (1.6) by the matrix of local turn

$$L^{(0)}_{(0)}=L^{(3)}_{(3)}=1, \quad L^{(1)}_{(1)}=L^{(2)}_{(2)}=\cos\Phi. \quad L^{(1)}_{(2)}=-L^{(2)}_{(1)}=-\sin\Phi$$

defined by the polar angle Φ in (1.4), we obtain the fixed pericentric tetrad

$$h^{*\mu}_{(0)}=(a,0,0,0), \quad h^{*\mu}_{(1)}=\left(0,\frac{\cos\Phi}{a},\frac{\nu\sin\Phi}{Hr},-\frac{h\sin\Phi}{Hr\sin^2\theta}\right)$$
$$h^{*\mu}_{(2)}=\left(0,\frac{\sin\Phi}{a},-\frac{\nu\cos\Phi}{Hr},\frac{h\cos\Phi}{Hr\sin^2\theta}\right), \quad h^{*\mu}_{(3)}=\left(0,0,-\frac{h}{Hr\sin\theta},-\frac{\nu}{Hr\sin\theta}\right) \tag{1.8}$$

with constant triad orientation coincident with the initial (orbital) orientation at the instant of passing the first pericenter of orbit ($\Phi=0$). Thus the invariability of vector $h_{(1)}{}^{*\mu}$ means that the invariant angle between the present radius vector and vector $h_{(1)}{}^{*\mu}$ is at any point (r,Φ) of the orbit equal to the polar angle Φ, while for vector $h_{(2)}{}^{*\mu}$ the respective angle is equal $\pi/2-\Phi$. The fixed tetrads (1.6) and (1.8) used here as auxilliary in the construction of accompanying RS are also useful in investigations of moments of external forces acting on a satellite (see, e.g., /4/).

To pass to an accompanying RS with the same triad orientation we apply the local Lorentz transform (local boost) of the form /1/

$$L^{(k)}_{(0)}=L^{(0)}_{(k)}=-u^{(k)}, \quad L^{(a)}_{(b)}=\delta^{(a)}_{(b)}+\alpha u^{(a)}u_{(b)}, \quad \alpha=(a\varepsilon+1)^{-1} \tag{1.9}$$

whose parameters are tetrad components of the 4-velocity reference body fixed in the RS (1.6) and (1.8)

$$u^{(k)}=h_\mu{}^{(k)}u^\mu=(a\varepsilon,\ A,\ H/r,\ 0)$$
$$u^{*(k)}=h^{*(k)}_\mu u^\mu=\left(a\varepsilon, A\cos\Phi-\frac{H}{r}\sin\Phi, A\sin\Phi+\frac{H}{r}\cos\Phi,0\right)$$

Having acted on (1.6) by transform (1.9) with parameters $u^{(k)}$, for the accompanying tetrad with the triad orbital orientation we obtain

$$h^\mu_{(0)}=\left(\varepsilon a^2,\frac{A}{a},-\frac{\nu}{r^2},\frac{h}{r^2\sin^2\theta}\right) \tag{1.10}$$
$$h^\mu_{(1)}=\left(aA,\frac{M}{a},-\frac{\alpha\nu A}{r^2},\frac{\alpha hA}{r^2\sin^2\theta}\right), \quad M=1+\alpha A^2$$
$$h^\mu_{(2)}=\left(\frac{aH}{r},\frac{\alpha HA}{ar},-\frac{\nu N}{Hr},\frac{hN}{Hr\sin^2\theta}\right), \quad N=1+\frac{\alpha H^2}{r^2}$$
$$h^\mu_{(3)}=\left(0,0,-\frac{h}{Hr\sin\theta},-\frac{\nu}{Hr\sin\theta}\right)$$

which for the equatorial motion of the reference body, after the transposition of the second and third columns, becomes the same as the "normal-diagonal" tetrad /9/. The application of transform (1.9) with parameters $u^{*(k)}$ to (1.8) yields for the accompanying pericentric tetrad

$$h^{*\mu}_{(0)}=\left(\varepsilon a^2,\frac{A}{a},-\frac{\nu}{r^2},\frac{h}{r^2\sin^2\theta}\right) \tag{1.11}$$
$$h^{*\mu}_{(1)}=\left(a\left(A\cos\Phi-\frac{H}{r}\sin\Phi\right),a^{-1}\left(M\cos\Phi-\frac{\alpha AH}{r}\sin\Phi\right)\right.$$
$$\left.\frac{\nu V}{Hr},-\frac{hV}{Hr\sin^2\theta}\right), \quad V=N\sin\Phi-\frac{\alpha AH}{r}\cos\Phi$$
$$h^{*\mu}_{(2)}=\left(a\left(A\sin\Phi+\frac{H}{r}\cos\Phi\right),a^{-1}\left(M\sin\Phi+\frac{\alpha AH}{r}\cos\Phi\right)\right.$$
$$\left.-\frac{\nu W}{Hr},\frac{hW}{Hr\sin^2\theta}\right), \quad W=N\cos\Phi+\frac{\alpha AH}{r}\sin\Phi$$
$$h^{*\mu}_{(3)}=\left(0,0,-\frac{h}{Hr\sin\theta},-\frac{\nu}{Hr\sin\theta}\right)$$

In the tetrad formalism the RS properties are determined by the dynamic characteristics expressed in terms of Ricci rotation coefficients $\gamma_{(k)(n)(m)}=h_{(k)}{}^\mu h_{(m)}{}^\nu\nabla_\nu h_{\mu(n)}$ /10/, where ∇_μ is the covariant derivative. The nonzero components $\gamma_{(k)(n)(m)}$ of tetrads (1.10) and (1.11) are of the form

$$\gamma_{(0)(1)(1)}=\frac{ma}{Ar^2}-\frac{H^2}{aAr^3}\left[1-\frac{ma^2}{r}(1-\alpha^2A^2)\right] \tag{1.12}$$
$$\gamma_{(1)(3)(3)}=\frac{\alpha AH^2\cos\theta}{\nu r^2\sin\theta}-\frac{M}{ar}$$

$$\begin{aligned}
&\gamma_{(0)(1)(2)} = \frac{H}{ar^3}\left(1-\frac{m\alpha Na^2}{r}\right), \quad \gamma_{(1)(2)(0)} = -\frac{H}{ar^2}\left(1-\frac{m\alpha\varepsilon a^3}{r}\right)\\
&\gamma_{(0)(2)(2)} = -\frac{A}{ar}\left(1-\frac{m\alpha^2H^2a^2}{r^3}\right), \quad \gamma_{(0)(3)(3)} = \frac{H^2\cos\theta}{\nu r^2\sin\theta} - \frac{A}{ar}\\
&\gamma_{(1)(2)(2)} = -\frac{\varepsilon}{r} - \frac{\alpha H^2}{ar^3}\left[1-\frac{ma^2}{r}(1+a\varepsilon\alpha)\right],\\
&\gamma_{(2)(3)(3)} = \frac{NH\cos\theta}{\nu r\sin\theta} - \frac{\alpha AH}{ar^2}\\
&\gamma_{(1)(2)(1)} = \frac{H}{aAr^2}\left[\frac{\alpha H^2}{r^4} - \frac{ma^2}{r}\left(1+\frac{H^2}{r^2} - N\alpha\varepsilon^2a^2\right)\right]\\
&\overset{*}{\gamma}_{(0)(1)(1)} = \cos^2\Phi\gamma_{(0)(1)(1)} + \sin^2\Phi\gamma_{(0)(2)(2)} - \sin 2\Phi\gamma_{(0)(1)(2)} \qquad (1.13)\\
&\overset{*}{\gamma}_{(1)(3)(3)} = \cos\Phi\gamma_{(1)(3)(3)} - \sin\Phi\gamma_{(2)(3)(3)}, \quad \overset{*}{\gamma}_{(0)(1)(2)} = \cos 2\Phi\gamma_{(0)(1)(2)} +\\
&\quad \cos\Phi\sin\Phi(\gamma_{(0)(2)(2)} - \gamma_{(0)(1)(1)}), \quad \overset{*}{\gamma}_{(1)(2)(0)} = \frac{H}{r^2} + \gamma_{(1)(2)(0)}\\
&\overset{*}{\gamma}_{(0)(2)(2)} = \cos^2\Phi\gamma_{(0)(2)(2)} + \sin^2\Phi\gamma_{(0)(1)(1)} + \sin 2\Phi\gamma_{(0)(1)(2)}\\
&\overset{*}{\gamma}_{(0)(3)(3)} = \gamma_{(0)(3)(3)}, \quad \overset{*}{\gamma}_{(1)(2)(2)} = \cos\Phi\left(\gamma_{(1)(2)(2)} - \frac{N}{r}\right) +\\
&\quad \sin\Phi\left(\gamma_{(1)(2)(1)} - \frac{\alpha AH}{r^2}\right)\\
&\overset{*}{\gamma}_{(2)(3)(3)} = \cos\Phi\gamma_{(2)(3)(3)} + \sin\Phi\gamma_{(1)(3)(3)},\\
&\overset{*}{\gamma}_{(1)(2)(1)} = \cos\Phi\left(\gamma_{(1)(2)(1)} - \frac{\alpha AH}{r^2}\right) - \sin\Phi\left(\gamma_{(1)(2)(2)} - \frac{N}{r}\right)
\end{aligned}$$

In the derivation of (1.13) the relation between Φ, ν and h implied by (1.7), was used. It follows from (1.12) and (1.13) that for the introduced RS the acceleration vector $F_{(a)} = \gamma_{(0)(a)(0)}$ and the angular rotation velocity tensor $A_{(a)(b)} = \gamma_{(0)[(a)(b)]}$ are zero, which shows that these RS are free falling and locally nonrotating, while the deformation rate tensor $D_{(a)(b)} = -\gamma_{(0)((a)(b))}$ is nonzero.

2. It was shown in /11/ that according to the GRT a gyroscope in orbital motion must have a geodesic precession of the vector of its angular momentum (spin). We shall assume that gyroscope spin does not disturbs the geodeticity of its motion. Then using Papapetru's equations /5,11/, we find that the gyroscope precession is determined in the accompanying RS by the equation

$$dS^{(a)}/dx^{(0)} = -\gamma^{(a)}_{(b)(0)}S^{(b)} = \Omega^{(a)}_{(b)}S^{(b)}$$

for the tetrad 3-vector spin $S^{(a)} = h_\mu{}^{(a)}S^\mu$.

The precession angular velocity

$$\Omega^{(a)} = \frac{1}{2}\varepsilon^{(a)(b)(c)}\Omega_{(b)(c)} = (\gamma_{(2)(3)(0)}, \gamma_{(3)(1)(0)}, \gamma_{(1)(2)(0)}) \qquad (2.1)$$

where $\varepsilon^{(1)(2)(3)} = \varepsilon^{(1)(2)(3)}_{(0)} = -1$), is not a 3-vector one relative to local three-dimensional turns, hence the quantity $\Omega = (\Omega^{(a)}\Omega_{(a)})^{1/2}$ depends on the triad orientation.

Tetrads with a triad tangent to coordinate lines were generally used in the calculation of (2.1) (see, e.g., /11–14/), and only approximate expressions for $\Omega^{(a)}$ or special types of orbits (circular or radial) were considered. An exact formula was obtained in /15/ for the precession angle, which in the particular case of the circular orbit differs from that given in /16/. Hence it is interesting to obtain an exact expression for the angular velocity and the precession angle of a gyroscope in the case of arbitrary orbits of form (1.3) and various triad orientation.

The determination of $\Omega^{(a)}$ in the accompanying orbital and pericentric RS with (1.12) and (1.13) taken into account yields for the single nonzero angular velocity component the expression

$$\Omega^{(3)} = -\frac{H}{ar^2}\left(1-\frac{m\alpha\varepsilon a^3}{r}\right), \quad \Omega^{*(3)} = \frac{H}{r^2} + \Omega^{(3)} \qquad (2.2)$$

After the substitution of ε and H defined in (1.5) for any arbitrary quasi-conical orbits of form (1.3), we obtain for the angular velocity of the gyroscope precession the following exact expressions in terms of functions of orbital parameters of its trajectory:

$$\Omega^{(3)} = \Omega^{*(3)} - \frac{\sqrt{m}w^2}{p\delta} = -\frac{\sqrt{m}\lambda w^2}{p^{3/2}\delta}\left[1-\frac{mw}{\lambda^2}\left(1+\frac{\lambda\delta}{\varkappa}\right)^{-1}\right], \quad \lambda = p - 2mw \qquad (2.3)$$

Taking into account $dx^{(0)} = h_\mu^{(0)} dx^\mu = ds$ and (1.2), we obtain for the precession angles during the time the gyroscope moves between points (r_1, Ψ_1) and (r_2, Ψ_2) of orbit (1.3) the expressions

$$\beta = \int \Omega^{(3)}\, dx^{(0)} = -\int_{\Psi_1}^{\Psi_2} \left(1 - \frac{m\alpha\varepsilon a^3}{p} w\right) \frac{d\Psi}{aF(\Psi)} \tag{2.4}$$

$$\beta^* = \int \Omega^{*(3)}\, dx^{(0)} = \int_{\Psi_1}^{\Psi_2} \frac{d\Psi}{F(\Psi)} + \beta = \Phi_2 - \Phi_1 + \beta \tag{2.5}$$

For a single turn of circular motion $(e = 0,\ p = R)$ we have

$$\beta = -2\pi\,(1 - 3m/R)^{1/2}, \quad \beta^* = 2\pi + \beta \tag{2.6}$$

where the expression for β^* is the same as the exact expression in /16/ derived by another method, while the respective approximate results agree with those in /11—14/. It follows from (2.2) that in the case of circular motion the quantity $\Omega^{*(3)}$ increases as the orbit radius decreases, and at the boundary of circular orbit existence $(R = 3m)$ becomes infinite, while $\Omega^{(3)}$ infinitely increases only when $R \to 0$.

The comparison of tetrads (2.4) and (2.5) shows that the pericentric orientation of the triad is preferable for analyzing gyroscope precession, since it makes possible the elimination of the "apparent" precession at angular velocity H/r^2 induced by the rotation of the orbital triad relative to the pericentric.

Using tetrads (1.10) and (1.11), we obtain for the physical components of an arbitrary electromagnetic field the following expressions in the form of functions of orbital parameters of the magnetometer trajectory:

$$E_{(1)} = NE_1 + \frac{\alpha a A\Delta_1}{r^2} - \frac{\Delta_4}{ar^2}, \quad H_{(1)} = \frac{N}{a^2} H_1 + \frac{a\Delta_3}{r^2} + \frac{\alpha A\Delta_2}{ar^2} \tag{2.7}$$
$$E_{(2)} = -\frac{\alpha AH}{r} E_1 - \frac{aM\Delta_1}{Hr} + \frac{A\Delta_4}{aHr},$$
$$H_{(2)} = -\frac{\alpha AH}{a^2 r} H_1 - \frac{aA\Delta_3}{Hr} - \frac{M\Delta_2}{aHr}$$
$$E_{(3)} = -\frac{H}{a^2 r} H_1 - \frac{\varepsilon a^2\Delta_3}{Hr} - \frac{A\Delta_2}{aHr}, \quad H_{(3)} = \frac{H}{r} E_1 + \frac{aA\Delta_1}{Hr} - \frac{\varepsilon\Delta_4}{Hr}$$

$$E^*_{(1)} = E_{(1)}\cos\Phi - E_{(2)}\sin\Phi, \quad H^*_{(1)} = H_{(1)}\cos\Phi - H_{(2)}\sin\Phi \tag{2.8}$$
$$E^*_{(2)} = E_{(1)}\sin\Phi + E_{(2)}\cos\Phi, \quad H^*_{(2)} = H_{(1)}\sin\Phi + H_{(2)}\cos\Phi$$
$$E^*_{(3)} = E_{(3)}, \quad H^*_{(3)} = H_{(3)}$$
$$\Delta_1 = \nu E_2 - \frac{hE_3}{\sin^2\theta}, \quad \Delta_2 = \nu H_2 - \frac{hH_3}{\sin^2\theta}$$
$$\Delta_3 = \frac{\nu E_3 + hE_2}{\sin\theta}, \quad \Delta_4 = \frac{\nu H_3 + hH_2}{\sin\theta}$$

Let us apply these expressions in the case of a dipole magnetic field which is a first approximation model of the Earth field, using the orbital RS. Contribution of the Schwarzschild metric to physical components of intensity $H_{(\alpha)}$, which is due to the use of tetrads, may prove to be comparable to the direct effect of phone metricon the solution of Maxwell's general-covariant equations. Hence it is reasonable to take into consideration that effect on the dipole magnetic field of the Earth. For the global components of the electromagnetic field tensor $F_{\mu\nu}$ in the spherical coordinate system, using the data of /17/, we obtain

$$F_{23} = \frac{r^2\sin\theta}{a^2} H_1 = \frac{2\mu\cos\theta\sin\theta}{r} f(r) \tag{2.9}$$
$$f(r) = \frac{3r^3}{4m^3}\left[\ln a - \frac{m}{r}\left(1 + \frac{m}{r}\right)\right]$$
$$F_{31} = \sin\theta H_2 = \frac{\mu a\sin^2\theta}{r^2} g(r), \quad g(r) = \frac{3r^2}{4m^2}\left(1 + a^2 - \frac{2r}{m}\ln a\right)$$

Substituting (2.9) into (2.7) we obtain the observed magnetic field components in the accompanying orbital RS

$$H_{(1)} = \frac{2\mu\cos\theta}{r^3} Nf(r), \quad H_{(2)} = -\frac{\mu\sin\theta\nu}{r^3 H} g(r), \quad H_{(3)} = -\frac{\mu\varepsilon ah}{r^3 H} g(r) \tag{2.10}$$

To compare (2.10) with formulas derived without allowance for the effect of gravitation on $H_{(\alpha)}$ and on RS, we express the respective intensity components in the "Newtonian" tetrad obtained from (1.6) with $a = 1$

$$H^0_{(1)} = \frac{2\mu\cos\theta}{r^3}, \quad H^0_{(2)} = -\frac{\mu\sin\theta\nu}{r^3 H}, \quad H^0_{(3)} = -\frac{\mu h}{r^3 H} \tag{2.11}$$

which coincides with the formulas obtained in /6/ with the use of matrix (1.7). Corrections due to the GRT (to the gravitation field and accelerated motion of the RS) are determined by the differences of (2.10) and (2.11)

$$\delta_1 = \frac{2\mu\cos\theta}{r^3}[Nf(r) - 1], \quad \delta_2 = \frac{\mu\sin\theta\nu}{r^3 H}[1 - g(r)], \quad \delta_3 = \frac{\mu h}{r^3 H}[1 - \varepsilon a g(r)] \tag{2.12}$$

and in the pericentric RS are of the same order of magnitude.

Assuming the dipole magnetic moment of the Earth to be $\mu = 8\cdot 10^{25}$ Hz.cm^3, we have from (2.12) that in the case of a satellite in circular orbit 500 km above the Earth surface the GRT corrections are $\sim 2\cdot 10^{-10}$ Hz, i.e. they are at the limit of modern magnetometers sensitivity. However in the case of an orbit of radius $4R_\odot$ around the Sun (as in the planned "solar probe" /18/) these corrections increase to $5\cdot 10^{-8}$ Hz, and can be measured by the high-precision quantum magnetometers that are intended for satellites in relativistic experiments /19/.

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